

# Super-Eddington Radiation Transfer in Soft Gamma Repeaters

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## ABSTRACT

Bursts from soft gamma repeaters have been shown to be super-Eddington by a factor of 1000 and have been persuasively associated with compact objects. Here, a model of super-Eddington radiation transfer on the surface of a strongly magnetic ( $\geq 10^{13}$  gauss) neutron star is studied and related to the observational constraints on soft gamma repeaters. In strong magnetic fields, the cross-section to electron scattering is strongly suppressed in one polarization state, so super-Eddington fluxes can be radiated while the plasma remains in hydrostatic equilibrium. The model offers a somewhat natural explanation for the observation of similarity between spectra from bursts of varying intensity. The radiation produced in the model is found to be linearly polarized to about 1 part in 1000 in a direction determined by the local magnetic field, and the large intensity variations between bursts are understood as a change in the radiating area on the source. Therefore, the polarization may vary as a function of burst intensity, since the complex structure of the magnetic field may be more apparent for larger radiating areas. It is shown that for radiation transfer calculations in this limit of super-strong magnetic fields it is sufficient to solve the radiation transfer equations for the low opacity state rather than the coupled equations for both. With this approximation, standard stellar atmosphere techniques are utilized to calculate the model energy spectrum.

*Subject Headings:* Gamma-Rays: Bursts – Radiation Processes: Thermal –

Stars: Neutron – X-Rays: Bursts

## I. INTRODUCTION

Soft gamma repeaters are a class of super-Eddington, repeating, high energy transients. The first repeating soft gamma ray bursters were recognized in the early eighties: SGR0526-22, (Mazets and Golenetskii 1981; Golenetskii, Iiyinskii, and Mazets 1984) and SGR1900+14 (Mazets, Golenetskii, and Guryan 1979). The discovery of over 100 repetitions of the third repeater, SGR1806-20 (Laros *et al.* 1986, 1987; Atteia *et al.* 1987; cataloged in Ulmer *et al.* 1993), secured the claim that soft gamma repeaters are a rare class of transients distinct from other high energy transients such as x-ray or gamma-ray bursts. Kouveliotou *et al.* (1993, 1994) have recently observed additional repetitions from the sources SGR1900+14 and SGR1806-20 with the Burst and Transient Source Experiment. Properties of SGRs are reviewed in Norris *et al.* (1991). Briefly, soft gamma-ray bursts typically last hundreds of milliseconds and have sharp rise and decay times (Atteia *et al.* 1987). They have no simple, discernible pattern of recurrence although they are clustered in time (Laros *et al.* 1987). A typical photon energy is 20–30 keV, and there is a strong rollover below about 15 keV (Fenimore, Laros, and Ulmer 1994). The sharp rise times as well as the eight second periodicity detected in the March fifth event (e.g. Mazets *et al.* 1979) from SGR0526-22 suggest that the sources are compact objects.

There has been much speculation as to the distance of the repeaters (e.g. Kouveliotou *et al.* 1987; Norris *et al.* 1991), particularly because SGR0526-22 is located in the direction of the Supernova remnant N49 in the Large Magellanic Cloud (Evans *et al.* 1980) with a very small error box of  $0.1 \text{ arcmin}^2$  (Cline *et al.* 1982). Recent evidence strongly favors a Population I distribution. Murakami *et al.* (1994) corroborated the association between SGR1806-20 and a plerionic supernova

remnant (Kulkarni and Frail 1993, hereafter KF) by fortuitously imaging a soft gamma ray burst with the ASCA x-ray satellite and thereby localizing the burster to within  $\sim 8 \text{ arcmin}^2$  — an improvement by a factor of 50 from the previous gamma-ray localization.

The distance to the supernova remnant has been estimated to be 17 kpc (KF), but, as KF point out, the distance method (surface-brightness/diameter), can have large errors (factors of 2 or 3) due to fluctuations in the local density and magnetic field (e.g. Miln 1979). In any case, it is likely that the source is at least 5 kpc away. Fenimore, Laros, and Ulmer (1994) (hereafter FLU), find that the x-ray spectrum of the bursts from SGR1806-20 has a sharp rollover below 15 keV and can therefore estimate the total flux of an individual burst by way of analysis of the spectra of 95 soft gamma-ray bursts detected by the *International Cometary Explorer* (ICE). A lower bound on the flux for the brightest events, assuming a distance of 5 kpc, is  $\sim 1.6 \times 10^{41} \text{ erg sec}^{-1}$ ,  $\sim 2 \times 10^3$  times the Eddington limit for a neutron star, and if at a distance of 17 kpc the flux would be larger by a factor of 10. The repeating bursts from SGR0526-22 were also super-Eddington by a large factor of  $1 - 2 \times 10^4$  assuming a location in the Large Magellanic Cloud. Furthermore, while the intensity of the individual bursts from SGR1806-20 has been detected to vary by over a factor of 50 (Laros, *et al.* 1987), the shape of the energy spectrum is remarkably constant above  $\sim 30 \text{ keV}$  and appears to be so at lower energies, too (FLU).

Therefore, the main points that need to be addressed in any radiation mechanism are that in the context of compact objects, the mechanism operate at highly super-Eddington fluxes, that it produce a low-energy roll-over, and that it produce a similar spectral shape over a wide range of intensities. Many radiation mechanisms which are partially successful with regard to these constraints are

discussed in FLU. In §II, the known mechanisms for producing super-Eddington fluxes are briefly discussed. A model which addresses the afore mentioned points is presented in §III. Energy spectra and radiation pressure produced by the model are calculated in §IV. Lastly, §V contains a discussion of the implications with regard to observations.

## II. Super-Eddington Fluxes

The problem of radiating super-Eddington fluxes from a compact object has been addressed in only a few different ways. The Eddington flux level is given by

$$L_E \sim \frac{4\pi GMm_H c}{\sigma_{Th}} \text{ erg sec}^{-1} \quad (1)$$

where  $G$  is the gravitational constant,  $M$  is the mass of the star,  $m_H$  is the mass of a proton,  $c$  is the speed of light, and  $\sigma_{Th}$  is the Thompson cross section. In general, there are three paths to tread with regard to the super-Eddington flux problem: (1) in some circumstances, the physical cross-section,  $\sigma$ , is reduced and the Eddington limit increases so seemingly super-Eddington fluxes can be radiated while maintaining hydrostatic equilibrium (2) the restraining force on the matter is increased above that of gravity alone, for instance, by magnetic pressure, so that the Eddington limit is increased, or (3) the matter is blown away at relativistic speeds, and a super-Eddington “fireball” is formed in front of it.

The first scenario, which is elaborated in §III, has been considered by Paczyński (1992), who observed that super-Eddington fluxes may be achievable in super strong magnetic fields (e.g. Thompson & Duncan 1993) where the Thompson cross-section is suppressed in one polarization state (Herold 1979).

The second scenario has primarily been considered with respect to gamma-ray burst models (e.g. Lamb 1982) and generally relies on magnetic pressure to confine

the radiating plasma. For blackbody emission, the required field is

$$B_{12} > \left[ \frac{T}{170 \text{ keV}} \right]^2.$$

While the typical temperatures of soft gamma-ray bursts are low enough that magnetic pressure can counteract the radiation pressure, the magnetic pressure acts perpendicular to the magnetic field, so matter is free to move along field lines and will at great speeds unless the magnetic field is exactly perpendicular to the radiation. Such a scenario then requires closed field line geometries. Also, for dipole fields, the combination of the radiation and magnetic pressures will tend to slide the radiating matter towards the equator and away from the star where the magnetic field is weaker. Such effects allow a smaller region to radiate and concentrate the matter at the weakest point in the magnetic fields. These general problems are significant and have yet to be addressed.

Finally, though never seriously considered in the context of soft-gamma repeaters, many fireball models of gamma-ray bursts are able to produce super-Eddington fluxes (e.g. Paczyński 1986, Goodman 1986). The main difficulties in adapting the fireball ideas to a scenario involving the soft gamma-ray repeaters stems from the fact that the photon energies are strongly peaked, quite low, and relatively uniform. As shown by FLU, over a large range of burst intensities, the hardness ratio is relatively constant. This means that for a radiation scenario similar to a blackbody, such as a thermal fireball model, the area of the fireball must change while maintaining a near constant temperature. Furthermore, parameters need to be somewhat finely tuned in order that the energy emerge from a fireball in radiative rather than kinetic energy (Rees & Mészáros 1992). The required fine tuning as well as the problem of changing the area of a fireball without changing the temperature make such scenarios appear unlikely, though they have not yet been investigated in

detail.

### III. Model of Radiation Transfer

As discussed above, a radiation mechanism for soft-gamma repeaters must address the super-Eddington flux problem, provide a strong roll-over in the data, and be able to produce a range of intensities without appreciatively changing the energy spectrum. If the sources for soft gamma-ray bursts are neutron stars with strong magnetic fields of order  $10^{13} - 10^{14}$  gauss, then all of these conditions can be met. In particular, hydrostatic equilibrium can be maintained as a result of a magnetically suppressed cross-section. A sharp rollover is produced which results from both the self absorption of the radiating plasma and the frequency dependent opacities. If the size of radiating surface varies, a range of intensities can be generated. Such variations might be produced by the deposit of a characteristic energy density below the surface which may occur during glitches or starquakes (e.g. Epstein 1992 and references therein). The surface temperature produced by such an energy release would be a slowly changing function of the depth of release, so that only a small range of surface temperatures would be observed, in accord with the observations. The energy is assumed to be released at large optical depth. For a neutron star, this condition is met if the energy release occurs more than about a centimeter below the neutron star surface for the low opacity state.

In strong magnetic fields, photon propagation becomes a strong function of polarization state (e.g. Herold 1979). In particular, since the motion of electrons is restricted perpendicular to the magnetic field, the Thompson cross-section for photons with linear polarization  $E_{\perp}$  with respect to the magnetic field direction (that is, with the plane of the photon electric field perpendicular to the stellar magnetic field direction) is much reduced relative to the normal cross-section.

Herold (1979) gives the relations for the total cross sections as:

$$\sigma_{\text{Tot}}(\parallel) \approx \sigma_{Th} \left[ \sin^2 \theta + \left( \frac{\omega}{\omega_B} \right)^2 \cos^2 \theta \right] \quad (2)$$

$$\sigma_{\text{Tot}}(\perp) \approx \sigma_{Th} \left( \frac{\omega}{\omega_B} \right)^2 \quad (3)$$

where  $\omega$  is the photon frequency,  $\omega_B \approx 10B_{12}$  keV is the electron cyclotron frequency in the magnetic field and  $\theta$  is the angle between the magnetic field and the Poynting vector. This equation is a good approximation when  $\omega$  is less than  $\omega_B$  and larger than  $\omega_B/1836$ , the proton cyclotron frequency. In superstrong magnetic fields,

$$\frac{\omega_B}{\omega_{\text{Teff}}} \approx 20 - 200 \quad (4)$$

where,  $\omega_{\text{Teff}}$  is the frequency corresponding to the effective temperature of soft gamma-ray bursts (5-10 keV generally), so that the  $E_\perp$  state has a lower opacity by  $400 - 4 \times 10^4$ . In this regime, there is also a simple relation between the differential cross-sections:

$$\frac{\sigma(\perp \rightarrow \perp)}{\sigma(\perp \rightarrow \parallel)} = 3 \quad (5)$$

$$\frac{\sigma(\parallel \rightarrow \parallel)}{\sigma(\parallel \rightarrow \perp)} \sim \left( \frac{\omega_B}{\omega} \right)^2. \quad (6)$$

Figure 1 shows a schematic of the radiation transfer under these circumstances. Due to the lower cross section in the  $E_\perp$  state, the mean free path is much longer and energy is transferred to the surface primarily in this state. Occasionally, a photon will scatter between states as shown by Eqs. 5,6. Because of the short timescales on neutron star surfaces (e.g. light crossing time of 0.1 milliseconds and Alfvén speed order  $c$ ) and the comparatively long durations of soft gamma-ray bursts ( $\sim 0.5$  seconds), the matter and two radiation states have ample time to achieve local thermal equilibrium. An additional requirement for LTE is a photon



generating radiation process (so that a Wein peak is not formed) which is not met by the electron scattering processes alone; however there are a number of second order processes such as double Compton scattering, proton cyclotron emission, one-dimensional thermal bremsstrahlung, and photon splitting, which can create photons.

Therefore, the system is in LTE, and the energy in the system is transferred primarily in the  $E_{\perp}$  state. The energy transfer can be quantified by examining the flux in the diffusion limit (e.g. Mihalas 1978):

$$\frac{F_{\nu}}{4\pi} \rightarrow \frac{1}{3} \frac{\partial B_{\nu}}{\partial \tau_{\nu}} = -\frac{1}{\sigma_{\nu}} \frac{1}{3} \frac{\partial B_{\nu}}{\partial T} \frac{dT}{dz}, \quad (7)$$

where  $\sigma_{\nu}$  is the cross-section as a function both of frequency and polarization state. In local thermal equilibrium, the ratio of the fluxes is given by

$$\frac{F(E_{\parallel})}{F(E_{\perp})} \approx \frac{\sigma_{\text{Tot}}(\perp)}{\sigma_{\text{Tot}}(\parallel)} \approx \left( \frac{\omega_{\text{B}}}{\omega} \right)^2 \approx 10^3 - 10^4. \quad (8)$$

The radiation pressure in a static medium is

$$P = \frac{1}{c} \int \sigma_{\nu} F d\nu, \quad (9)$$

therefore, the pressure from the  $E_{\perp}$  state photons is equal to that from the  $E_{\parallel}$  state. The sum of the pressures is less than what would be found in a non-magnetized system by roughly a factor of  $0.5 \times (\omega_{\text{B}}/\omega)^2$ , so the effective Eddington limit is much higher (Paczynski 1992).

To calculate the emergent energy spectrum from such a highly magnetized source, it suffices to follow the flow of radiation in the low opacity state. In particular, the cross section in the  $E_{\perp}$  state can be separated into a scattering component,  $\sigma_{\text{s}}$ , where the photon scatters into the  $E_{\perp}$  state and an absorption component, where the photon scatters into the  $E_{\parallel}$  state. The differential cross

sections in the  $E_{\parallel}$  states (Eqs. 5,6) are such that a photon will scatter numerous times within the high opacity state, thermalize due to the photon production processes described above and eventually scatter back into the low opacity state. Double Compton scattering likely plays the dominate role in the thermalization (photon-production) because it is expected to be roughly a factor of ( $\alpha \approx 1/137$ ) smaller than the magnetically suppressed Thompson cross-section of Eq. 2 and a photon will likely scatter a thousand or more times before scattering out of the high opacity state.

### Model Calculations and Results

With the model described above, the energy spectrum can be calculated with regard only to the low opacity polarization state, so that the problem reduces to the calculation of a unpolarized stellar atmosphere with cross-sections for scattering and absorption determined by the suppressed Thompson scattering relations. Because the cross-section is a smooth function of frequency and the scattering and absorption portions of the cross-section are comparable, many methods are available for the calculation of the emergent spectra. Here, an iterative solution to the one-dimensional radiation transfer equation is utilized (e.g. Mihalas 1978 (6-1)) and the requirement of conservation of flux is met to within 2 percent with iteratively correcting the temperature as a function of optical depth using a procedure developed by Lucy (Lucy 1964; Mihalas 1978 (7-2)). The use of angle averaged quantities the iteration procedure in general is a good approximation, however, the magnetic field direction introduces a small anisotropy in the cross-sections, for which a more detailed calculation should account. Note however that the scattering/absorption ratio (Eq. 5) is angle independent.

The spectrum resulting from solving the radiation transfer equations is shown

in figure 2. The emergent flux is strongly altered from a blackbody. The peak flux occurs at twice the effective temperature ( $2/3$  that of a blackbody). The spectrum is primarily a function of  $T_{\text{eff}}$ . There is a much weaker dependence on  $\omega_B/\omega$  (100 in figure 2) when the ratio is in the range of 20-200 since this primarily scales the cross-section without changing its shape. Therefore, the spectral shape is a good approximation over a range of  $\omega_b/\omega$ .

Using the spectrum obtained from the radiation transfer model, the radiation pressure can be calculated and characterized as a function of peak flux energy:

$$P(E_{\perp}) = \frac{1}{c} \int \sigma_{\nu} F_{\nu}(E_{\parallel}) d\nu \approx \frac{2.9}{c} \sigma_{\text{Th}} \sigma_B T_{\text{eff}}^4 \frac{\omega_{\text{Teff}}}{\omega_B} \quad (10)$$

where  $\sigma_B$  is the Stefan-Boltzmann constant. In the diffusion limit, where the radiation pressure will generally be greatest, the high opacity state contributes approximately the same radiation pressure as the low opacity. Therefore, the Eddington limit in this regime, expressed as a function of the non-magnetic Eddington limit is:

$$L_{\text{Edd}} \approx \frac{1}{6.8} \left[ \frac{\omega_{\text{Teff}}}{\omega_B} \right]^2 \approx 60 - 6000 L_{\text{Edd}}. \quad (11)$$

## Discussion

Using the spectral shape derived above, the radiating area of the source can be estimated. For the events from SGR1806-20 for which complete x-ray spectra were determined (FLU), the peak flux occurs between about 8 and 20 keV. The effective temperature is found to be half of the frequency of peak flux, so the projected active source area for the brightest burst from SGR1806-20 is

$$A = \frac{D^2 L_{\text{measured}}}{\sigma_B T_{\text{eff}}^4} \approx 2 \times 10^3 \text{ km}^2 \left( \frac{D}{10 \text{ kpc}} \right) \left( \frac{T_{\text{eff}}}{7 \text{ keV}} \right)^{-4}, \quad (12)$$

where the uncertainties in  $T_{\text{eff}}$  and  $D$  can sway the area by about a factor of 20 in either direction. The maximum projected radiating area for a neutron star is approximately  $\pi r^2 \approx 300 \text{ km}^2$ , which is well within the error bars on the determined area. However, if the distance is much greater than 10 kpc and the effective temperature is less than about 10 keV, it may be difficult to reconcile the model with observations. Blackbody models would generally require even larger areas ( $\sim 5\text{--}10$ ) because for a blackbody,  $T_{\text{eff}} \approx T_{\text{peak}}/3$ . One obtains similar results for the bursts from SGR0526-22. The distance to this source is known with some certainty to be about 55 kpc; however, there were no x-ray observations below 30 keV, so it is not possible to determine where the frequency of peak flux occurs for this burster.

The radiation transfer model offers a somewhat natural explanation for the similarity between spectra of bursts with different intensities. If a characteristic energy density, or temperature, is deposited at large optical depth,  $\tau$ , within the star, the resulting surface temperature would only be a weak function of  $\tau$ . At large optical depth, the temperature relation for the grey atmosphere limit becomes a reasonable approximation, so

$$T^4 = \frac{3}{4} T_{\text{eff}}^4 [\tau + q(\tau)], \quad (13)$$

where  $q(\tau)$  is a number of order unity. Inverting the equation shows that for releases of constant energy density, so that  $T$  is constant, the effective surface temperature goes as the fourth root of the optical depth of release,  $\tau$ . However, as Thompson and Duncan (1992) observed, the suppressed opacity rises with temperature and therefore with optical depth. If the energy is released quite deep in the crust, the radiation pressure may be higher than the Eddington limit if, for example, the typical temperature is near the electron cyclotron energy. However, at such large optical depth, pressure from the matter above could offer a sufficient restraining

force to prevent expansion.

Beyond meeting the current observational constraints, this radiation transfer model predicts that SGR bursts should be locally linearly polarized to approximately 1 part in 1000, because the flux is dominated by the low opacity state. Polarization may then be a function of burst intensity, since the destruction of the polarization pattern by the global structure of the magnetic field become more apparent for larger intensities and areas. Second, if the variations in intensity of the soft gamma-ray bursts are, indeed, a result of varying sizes the “active” region, then there should be a critical intensity reached when the entire source is active.

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## FIGURE CAPTIONS

**Fig. 1:** This schematic illustrates the major properties of the radiation transfer in a strongly magnetized medium. The electron scattering cross-sections for photons differ by about a factor of 1000 between the two photon polarization states  $E_{\perp}$  and  $E_{\parallel}$  (plane of the photon electric field perpendicular or parallel to the magnetic field direction). Consequently, the flux in the low opacity,  $E_{\perp}$  state is much higher, and one can see much deeper into the star in this state.

**Fig. 2:** The energy spectra produced in a strongly magnetized stellar atmosphere is compared to a blackbody. The spectra produced by the strongly magnetized plasma peaks at a lower frequency and has a slightly faster fall off at high energies. The shift to lower energies is produced because the cross-section goes as the square of the frequency, so that at low frequencies the cross-section is lower, and one can see farther into the source where the temperatures are higher.



fig. 1

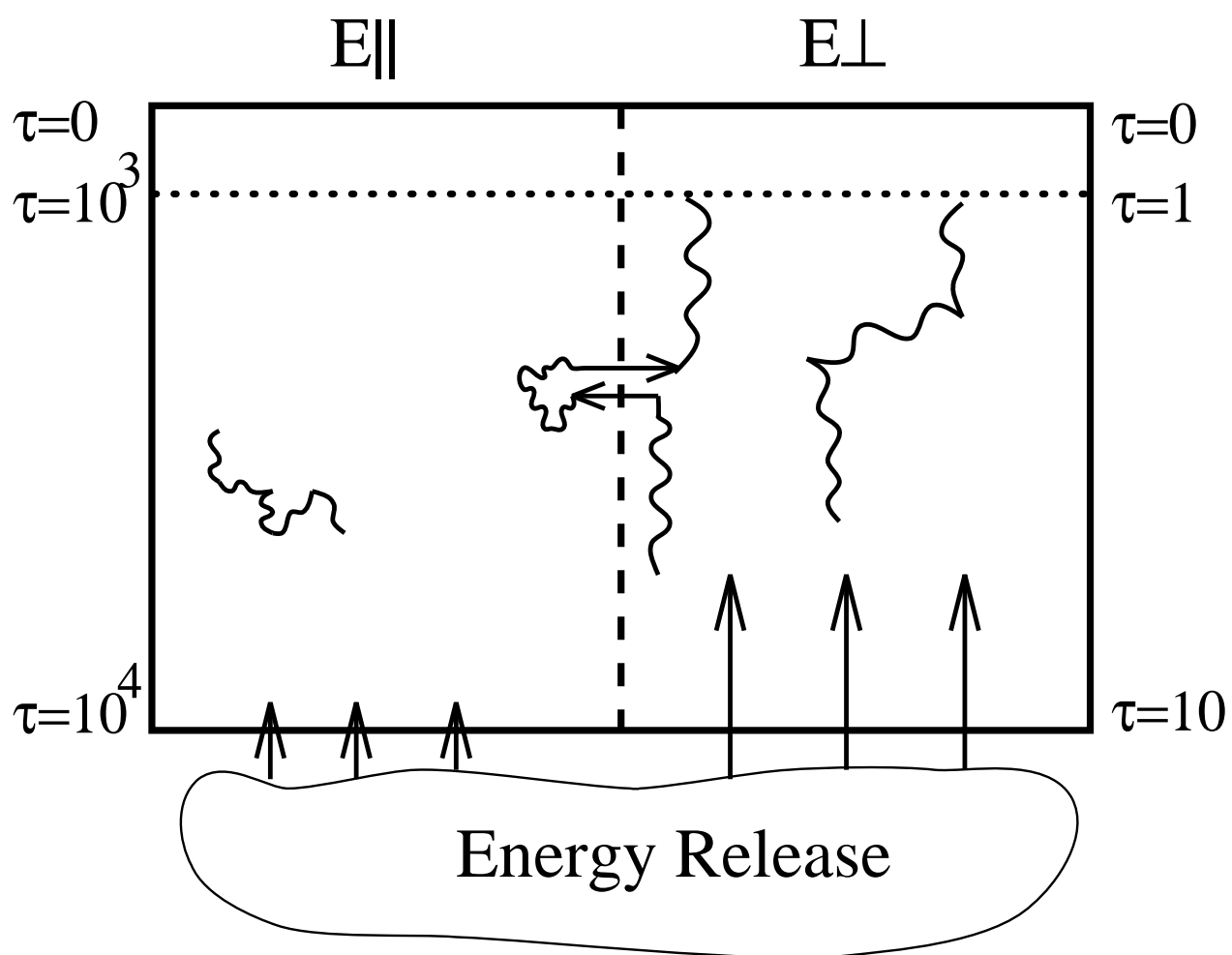


fig. 2

